BIJU PATNAIK UNIVERSITY OF TECHNOLOGY,
ODISHA

Lecture Notes

On

Markov Chain

Part 3

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Markov Chain Part 3

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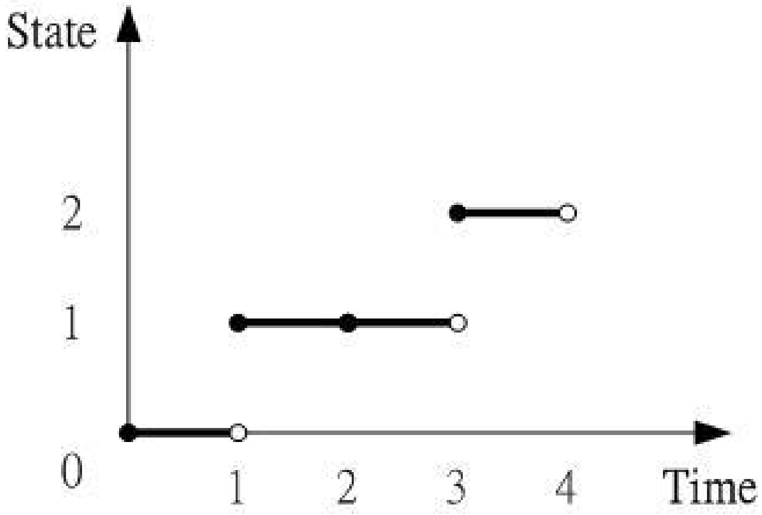
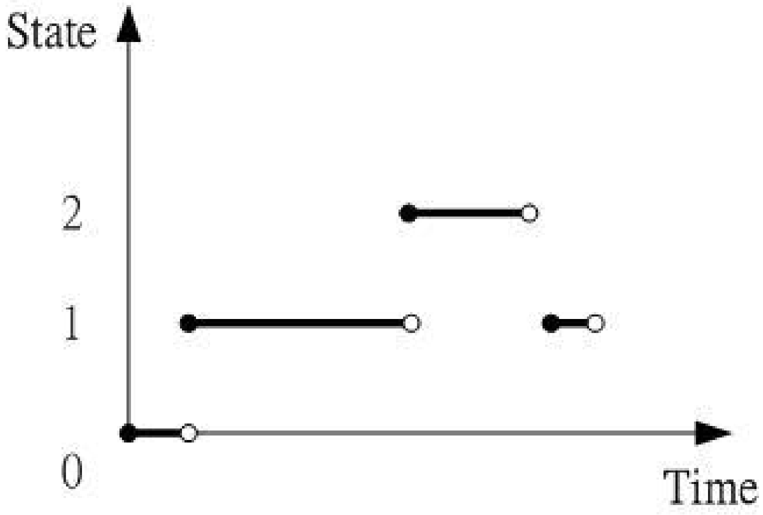
Outline

- Continuous Time Markov Chains
- An Example

Continuous Time Markov Chains

- Discrete Time V.S. Continuous Time





Continuous Time Markov Chains (cont.)

- $X(t')$: the state of the system at time t'

- Three points in time :
  - $t' = r$ is a past time
  - $t' = s$ is the current time
  - $t' = s+t$ is t units of time into the future

- Markovian property :
  - $P\{ X(s+t) = j \mid X(s) = i \text{ and } X(r) = x(r) \} = P\{ X(s+t) = j \mid X(s) = i \}$
    for all $i, j = 0, 1, 2, \ldots, M$ and for all $r \geqq 0$, $s > r$, and $t > 0$
  - $P\{ X(s+t) = j \mid X(s) = i \}$ is a transition probability.

Continuous Time Markov Chains (cont.)

- Stationary transition probability :
  - If the transition probabilities are independent of s, so that

    $P\{ X(s+t) = j \mid X(s) = i \} = P\{ X(t) = j \mid X(0) = i \}$

    they are called stationary transition probability.
  - $p_{ij}(t) = P\{ X(t) = j \mid X(0) = i \}$ is called the continuous time transition probability function.

- Assumption :

$$\lim_{t \to 0} p_{ij}(t) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Continuous Time Markov Chains (cont.)

- One key set of random variables, T_i :
  - Each time the process enters state i, the amount of time it spends in that state before moving to a different state. (i = 0, 1, 2, …, M)

- Memoryless :
  - $P\{ T_i > t + s \mid T_i > s \} = P\{ T_i > t \}$

Continuous Time Markov Chains (cont.)

- An equivalent way of describing a continuous time Markov chain :
  - The random variable T_i has an exponential distribution with a mean $1/q_i$.
  - P_{ij} : the probability of moving from state i to state j.

    $P_{ii} = 0$ and $\sum_{j=0}^{M} p_{ij} = 1$ for all i
  - The next state visited after state i is independent of the time spent in state i.

Continuous Time Markov Chains (cont.)

- Transition rates :
  - $q_i = \lim_{t \to 0} \frac{1 - p_{ii}(t)}{t}$
  - $q_{ij} = q_i p_{ij}$
- Steady-state probabilities
  - If a Markov chain is irreducible, then $\lim_{t \to \infty} p_{ij}(t) = \pi_j$
  - $\pi_j = \sum_{i=0}^{M} \pi_i p_{ij}(t)$ for j = 0, 1, 2, …, M

Continuous Time Markov Chains (cont.)

- Steady-state equation :

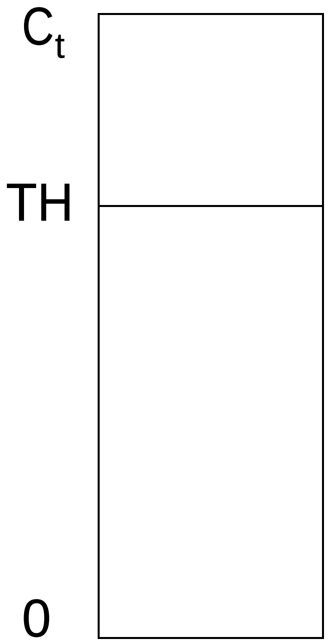
$$\pi_j q_j = \sum_{i \neq j} \pi_i q_{ij} \qquad \text{for } j = 0, 1, 2, \ldots, M$$

$$\sum_{j=0}^{M} \pi_j = 1$$

An Example

- Model the traditional guard-channel scheme using continuous time Markov channel.

- The tradition guard-channel scheme :



A new call is admitted only when there are less than TH channels occupied.

A handoff request is rejected only when all channels are occupied.

An Example (cont.)

- The system : A cell
- The state : the number of occupied channels in a cell

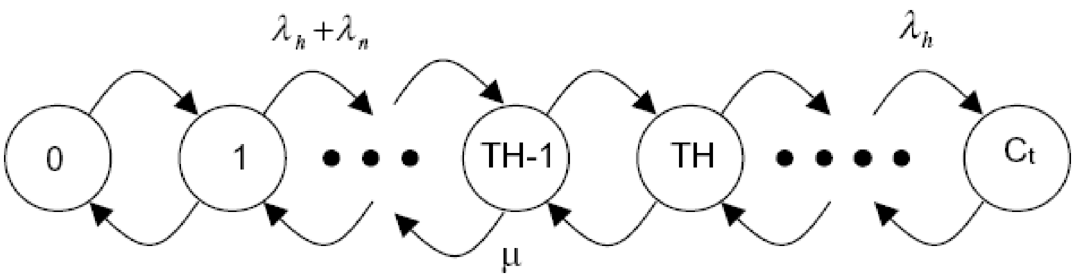


Fig. 2. Markovian model.

An Example (cont.)

- Steady-state probabilities :

$$p_i = \begin{cases} \frac{1}{i!}\left(\frac{\lambda_n + \lambda_h}{\mu}\right)^i p_0, & 0 < i \leqslant \mathrm{TH} \\ \frac{1}{i!}\left(\frac{\lambda_n + \lambda_h}{\mu}\right)^{\mathrm{TH}} \left(\frac{\lambda_h}{\mu}\right)^{i-\mathrm{TH}} p_0, & \mathrm{TH} < i \leqslant C_t \end{cases}$$

$$p_0 = \left(\sum_{i=0}^{\mathrm{TH}} \frac{1}{i!}\left(\frac{\lambda_n + \lambda_h}{\mu}\right)^i + \sum_{i=\mathrm{TH}+1}^{C_t} \frac{1}{i!}\left(\frac{\lambda_n + \lambda_n}{\mu}\right)^{\mathrm{TH}} \left(\frac{\lambda_h}{\mu}\right)^{i-\mathrm{TH}} \right)^{-1}$$

An Example (cont.)

- Call dropping probability :

$$p_{\mathrm{d}} = \frac{1}{C_{\mathrm{t}}!}\left(\frac{\lambda_{\mathrm{n}}+\lambda_{\mathrm{h}}}{\mu}\right)^{\mathrm{TH}}\left(\frac{\lambda_{\mathrm{h}}}{\mu}\right)^{C_{\mathrm{t}}-\mathrm{TH}} \cdot \left(\sum_{i=0}^{\mathrm{TH}}\frac{1}{i!}\left(\frac{\lambda_{\mathrm{n}}+\lambda_{\mathrm{h}}}{\mu}\right)^{i} + \sum_{i=\mathrm{TH}+1}^{C_{\mathrm{t}}}\frac{1}{i!}\left(\frac{\lambda_{\mathrm{n}}+\lambda_{\mathrm{h}}}{\mu}\right)^{\mathrm{TH}}\left(\frac{\lambda_{\mathrm{h}}}{\mu}\right)^{i-\mathrm{TH}}\right)^{-1}$$

- Call blocking probability :

$$p_{\mathrm{b}} = \sum_{i=\mathrm{TH}}^{m}\frac{1}{i!}\left(\frac{\lambda_{\mathrm{n}}+\lambda_{\mathrm{h}}}{\mu}\right)^{\mathrm{TH}}\left(\frac{\lambda_{\mathrm{h}}}{\mu}\right)^{i-\mathrm{TH}} \cdot \left(\sum_{i=0}^{\mathrm{TH}}\frac{1}{i!}\left(\frac{\lambda_{\mathrm{n}}+\lambda_{\mathrm{h}}}{\mu}\right)^{i} + \sum_{i=\mathrm{TH}+1}^{m}\frac{1}{i!}\left(\frac{\lambda_{\mathrm{n}}+\lambda_{\mathrm{h}}}{\mu}\right)^{\mathrm{TH}}\left(\frac{\lambda_{\mathrm{h}}}{\mu}\right)^{i-\mathrm{TH}}\right)^{-1}$$

An Example (cont.)

- Find an TH which guarantees that CDP is kept below the tolerable level.

$$p_{\mathrm{d}} = \frac{1}{C_{\mathrm{t}}!}\left(\frac{\lambda_{\mathrm{n}}+\lambda_{\mathrm{h}}}{\mu}\right)^{\mathrm{TH}}\left(\frac{\lambda_{\mathrm{h}}}{\mu}\right)^{C_{\mathrm{t}}-\mathrm{TH}} \cdot p_0 \leqslant p_{\mathrm{td}}$$

- Why not to keep CBP below the tolerable level ?

An Example (cont.)

- The proposed approach :
  - A cell is classified into two categories, hot cells and cold cells.
  - Hot cells : Cu > TH
  - Cold Cells : Cu ≦ TH
  - Cold cells follow the same CAC as in the traditional guard-channel scheme, while hot cells admit new calls with a probability, PCA, instead of blocking new calls absolutely.

$$\mathrm{PCA}(C_u) = \begin{cases} \sin\left(\left(\frac{C_u - \mathrm{TH}}{C_t - \mathrm{TH}}\right) \cdot \frac{\pi}{2} + \pi\right) + 1, & \text{if } \mathrm{TH} \leqslant C_u \leqslant C_t \\ 1, & \text{if } C_u < \mathrm{TH} \end{cases}$$

```
channel_allocation( )
{
  λh = the current arrival rate of handoff call;
  λn = the current arrival rate of new call;
  y = the tolerable dropping rate;
  Ct = the number of total channels;
  Cu = the number of occupied channels;
  TH = the value of threshold such that
```

$$p_d = \frac{1}{C_t!}\left(\frac{\lambda_n+\lambda_h}{\mu}\right)^{TH}\left(\frac{\lambda_h}{\mu}\right)^{C_t-TH}\cdot p_0 \leqslant p_{td};$$

```
  If (the arriving call is a handoff call)
    If (Ct − Cu > 0)
      Accept this call;
    Else
      Drop the handoff call;
  Else /* new call */
    If (Cu < TH)
      Accept this call;
    Else
```

$$F = \sin\left(\left(\frac{C_u-TH}{C_t-TH}\right)\cdot\frac{\pi}{2}+\pi\right)+1;$$

```
      r = random( );
    if (r < F)
      Accept this new call;
    else
      Block the new call;
}
```

An Example (cont.)

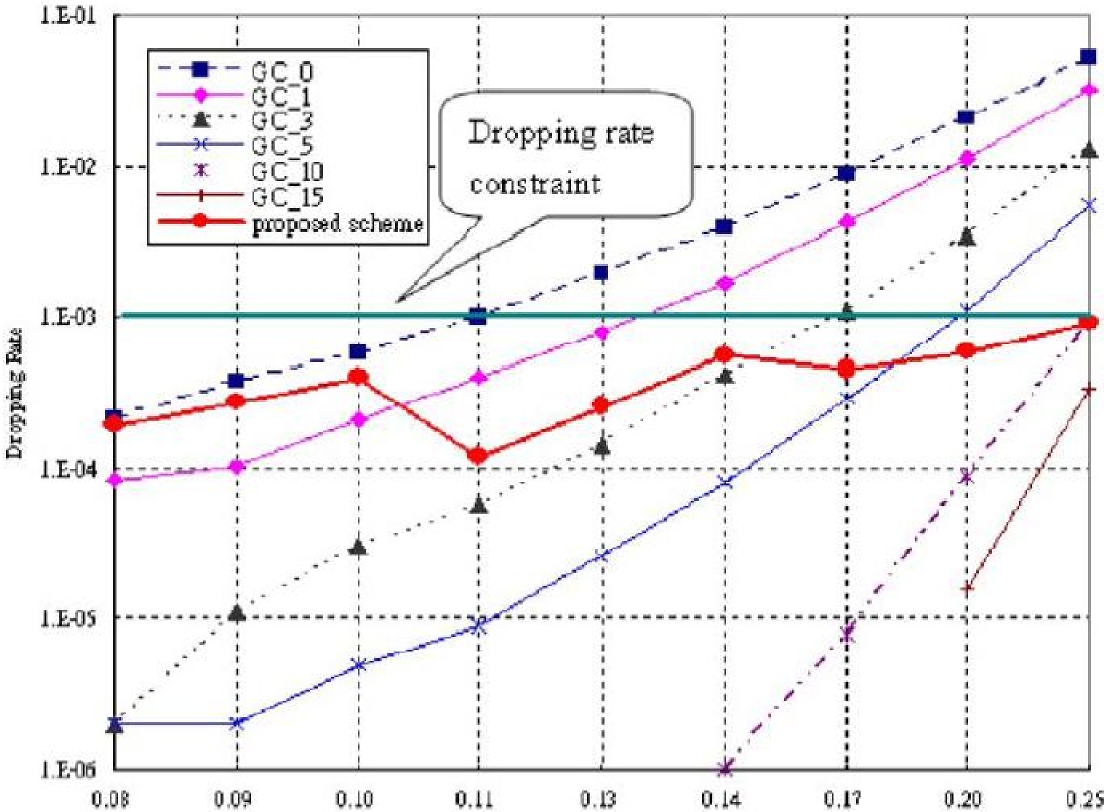


Fig. 4. The dropping rate with different arrival rate of handoff call ($\lambda_n = 1/6$).

An Example (cont.)

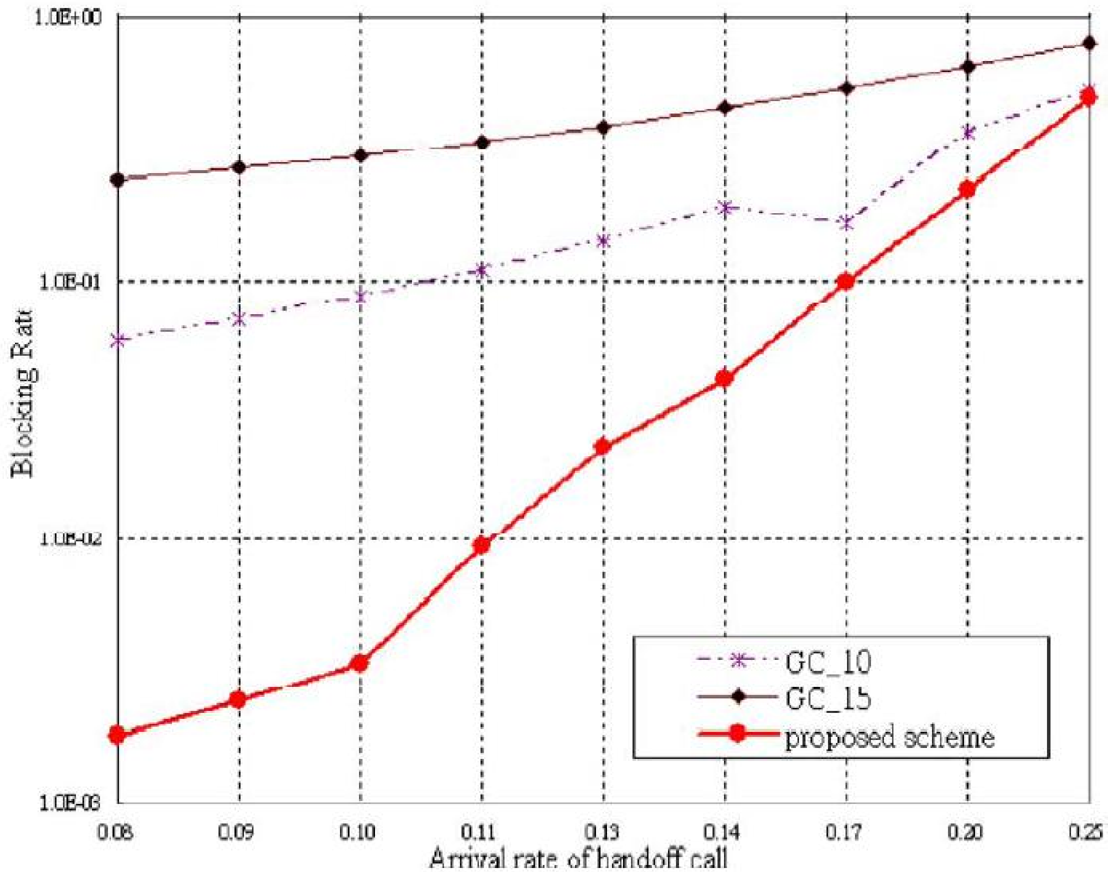


Fig. 5. Blocking rate versus arrival rate of handoff call (λ_n = 1/6).

Top Sentences

- Just as the transition probabilities for a discrete time Markov chain satisfy the Chapman Kolmogorov equations, the continuous time transition probability function also satisfies these equations.
- just as可用於比擬。 ---- I-Chi

- More specifically, a new call request is admitted only when there are less than TH channels occupied.
- more specifically可表示更進一步具體說明。 ---- I-Chi

- We shall restrict our consideration to continuous time Markov chains with the following properties.
- restrict our consideration to 可用於界定討論範圍。 ---- I-Chi

Reference

- *Hillier and Lieberman*, "Introduction to Operations Research", seventh edition, McGraw Hill

- *Jin-Long Wang and Shu-Yin Chiang*

  "Adaptive channel assignment scheme for wireless networks"

  Computers and Electrical Engineering

THANK YOU